Observation on the geometry behind the design of the 'Vitruvian man' by Leonardo da Vinci

Article · June 2018

1 author:

Leno Mascia
Loughborough University

Observations on the geometry behind the design of the 'Vitruvian Man' by Leonardo da Vinci

Leno L. Mascia

Loughborough University, Loughborough, UK.

1. Introduction

The drawing with the title *Proportions of the human body according to Vitruvius* by Leonardo da Vinci (1490), located in the *Gallerie dell' Accademia* in Venice, is a good example of the use of quantitative design parameters in the execution of works of art. Scholars from a variety of disciplines, particularly from the architects' community, have been drawn the attention to Leonardo's anthropometric parameters displayed by the man in the square and have debated on issues surrounding the golden section to determine the ratio of the side of the square (the height of the man in the standstill position) to the radius of the circle (the circumscription of the man in raised-arms and open-legs posture). Other important geometric features, such as the asymmetry inherent to the posture of the man in the circle, have so far not been addressed in an analytical and quantitative manner. From the very early stages of my research on Leonardo's Vitruvian man it became obvious that the literature had not paid attention to the 'oddity' of the posture of the legs. This feature became particularly attractive to me alongside the positioning of the extremities of the limbs inside the circle [1].

1. **Origin of the Vitruvian man**

In the *Ten Books on Architecture* (ca. 25 BC) the Roman architect Marcus Vitruvius Pollio draws attention to symmetry in the design of Temples (Book III, Chapter 1), which should reflect the proportionality of members of the human body represented by the forearm (*cubit*), the *palm* and the finger (*digit*) and the *foot*. Vitruvius states that the navel is naturally placed at the centre of the human body: ‘For if a man be placed flat on his back, with his hands and feet extended, and a pair of compass centred at his navel, the fingers and toes of his two hands and feet will touch the circumference of a circle described therefrom. And just as the human body yields a circular outline, so too a square figure may be found from it. For if we measure the distance from the soles of the feet to the top of the head, and then apply that measure to the outstretched arms, the breadth will be found to be the same as the height, as in the case of plane surfaces which are perfectly square’ [2]. The essence of the above statements is embedded in the delineating contours of the image in Leonardo’s drawing, alongside anthropometric indicators on the body of the man in the square. This iconic masterpiece has induced many followers to produce drawings portraying a man in square and in a circle, which became known as the ‘Vitruvian man’ [3]. In the upper annotations of Leonardo’s drawing there is a statement that has not been identified in the translations of Vitruvius’ *Ten Books*, which reads ‘If you open your legs so much as to decrease the height 1/14 and spread and raise your arms till the middle fingers touch the level of the top of your head you must know that the centre of the outspread limbs will be the navel and the space between the legs will be an equilateral triangle’ [4]. Leonardo’s Vitruvian man stands out from all other homonymous versions not only for its artistic qualities but also for introducing features, which suggest that the drawing may have been intended to be more than a mere illustration of Vitruvius canon for the proportionality of the members of the human body (coined *homo bene figuratus* in the Treatise on Painting) [5].

2. The Square, the Circle and the Golden Ratio in Leonardo’s Drawing

Leonardo’s assessment of the dimension and position of the square relative to the circle, as a design feature for the illustration of Vitruvius canon of human proportions related to the height of man and the spread of the arms, may have been inspired by the two drawings of Francesco di Giorgio Martini. The surprising aspect of the latter drawings is the attempt to depict the two postures of the man using a square concentric with the circle. Leonardo resolves the dilemma by separating the centre of the two figures, one for the circle centred at the navel and the other for the square centred at the pubic bone. Furthermore, he depicts the two postures in one single ‘diagram’ as a reflection of the scientific basis of the work, which derives from the experimental studies illustrated in Figure 1 (left). On the right side of Figure 1 is shown a comparison of the related image of these studies with Leonardo’s drawing of the Vitruvian man, using half sections to aid the eye in the assimilation of the two images. The position of the man kneeling in this image would have helped Leonardo to identify the 3/4 length of the body for the location of mark at the torso and the 1/4 length at the knee cap, while the man in the sitting position may have identified the midpoint at the pubic bone. The latter is another datum that does not feature in Vitruvius text and may be responsible for the dilemma faced by Francesco di Giorgio Martini in his attempt to make the two representations directly from Vitruvius statement in the original text (in Latin and devoid of any pictorial descriptions).

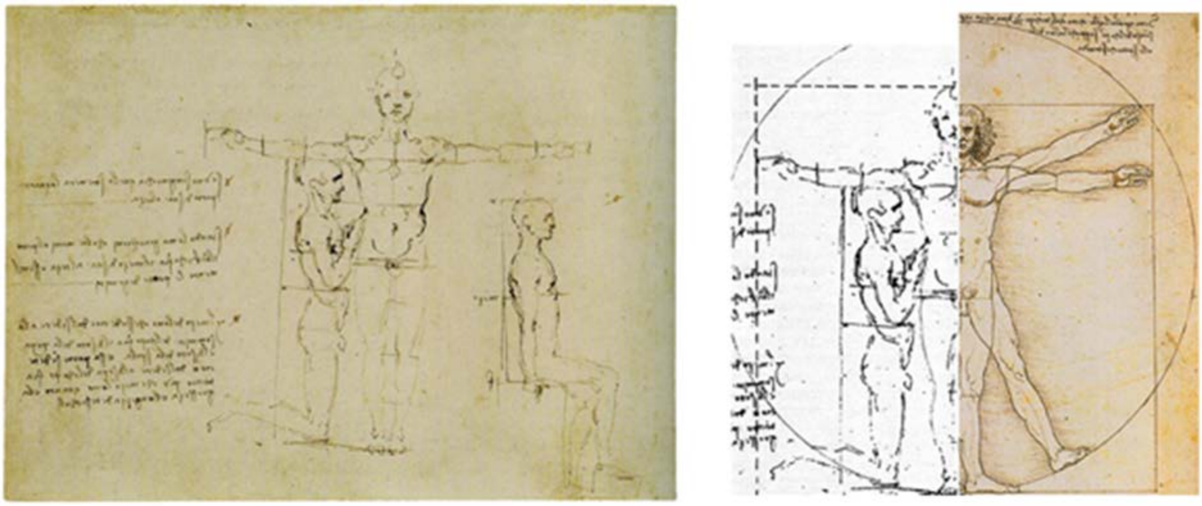

Figure 1: Left. Studies of the Proportions of the Body when Standing, Kneeling and Sitting RL 19132 r (c 1487), Windsor Castle. Right. Comparison of Leonardo's Vitruvian man his sketch of the man standing taken from the image on the left with the addition of the corresponding square and circle.

When a square with height and side equal to H and a circle with radius R (centred at the navel) are added to the image of the 'Man standing', the ratio H/R comes to 1.70 – 1.72. This is significantly higher than the 1.64 – 1.65 range identified in Leonardo's drawing (see comparison in Figure 1- right), and is substantially greater than the golden ratio value (φ = 1.618). Accordingly, the claims that Leonardo would have used the golden section to determine the dimensions of the square and the circle in circumscribing the man according to Vitruvius canon is likely to be based on expectation derived from either inaccurate measurements or approximations.

The reason why Leonardo would have not used a value for the H/R ratio closer to φ is rooted in the requirement for the finger tips in the raised arms position to be able to touch concurrently the circle and the square, while fulfilling the self-imposed stringent condition that these should be aligned with the crown of the head, as well as ensuring that the fingers retain the capability of contacting the lateral sides of the square in the stand-still posture. This interpretation is supported by the diagram in Figure 2 (right) by Manenti-Valli [6], which shows that if the radius of the circle were to be increased to reach an H/R ratio equal to 1.618 the fingers in the raised arms position would no longer be able to maintain the dual contact with the square and the circle. The same argument has been put forward by Takashi [7].

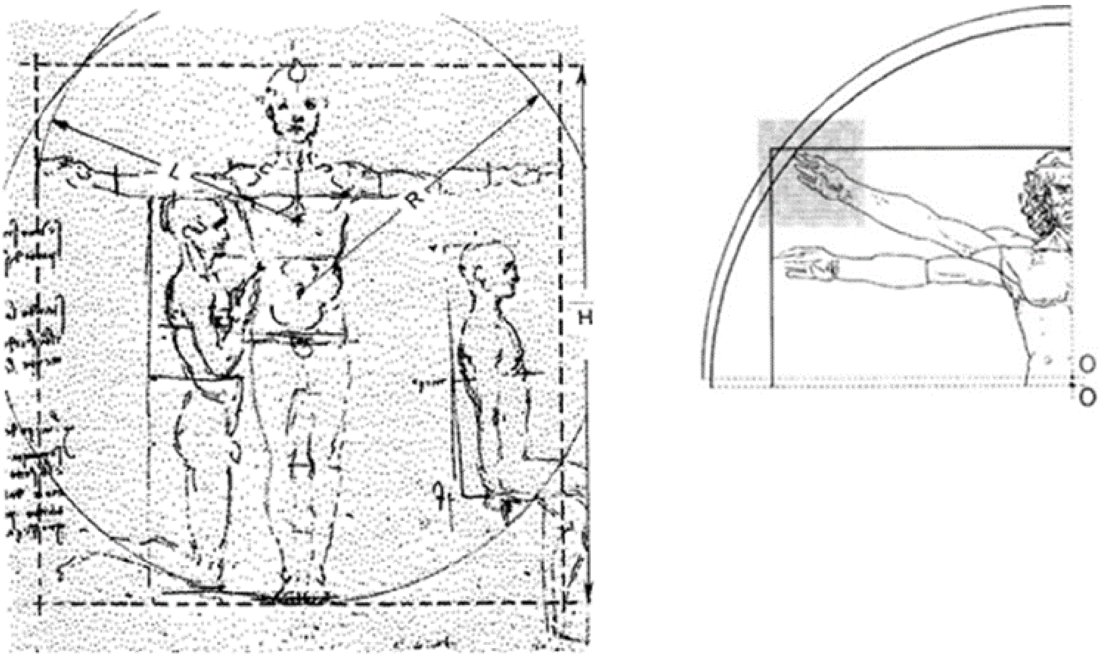

Figure 2. Left: Overlaid lines and symbols on the original sketch in Figure 1-left for the identification of relevant geometric features parameters. b) Right: Partial reproduction of image of the Vitruvian man from [6].

A reconstruction of the Vitruvian man drawing derived directly from the sketch of the 'Man standing' (Figure 2 - left) identifies the dimension and position of the vector (L) that would have been required to maintain the length consistency in rotations between the horizontal and angular positions. In this case the vector does not coincide with the arms and, therefore, would require a reduction in the height of the man to shift the rotation point upwards at the midpoint of the collarbone to satisfy the imposed conditions for the dual contact of the fingers with the circle and the square at the level of the crown of the head, as clearly indicated in Figure 2 -left. This adjustment has also led him to identify the altitude of the collarbone as a parameter of human proportions, which is recorded in the 9th line of the annotations as the distance from 'above the chest to the crown of the head' is equal to 1/6 the height of the man (see Endnote 1). It is worth noting that this anthropometric parameter corresponds to 1/5 in the sketch in Figure 1 (left) for Studies of the Proportions of the Body when Standing, Kneeling and Sitting RL 19132 r. The change in the numerical value, therefore, must have been made specifically to meet the condition for the man to fit contemporarily in both the square and the circle in the aligned configuration of the fingers with the crown of the head.

It is worth noting that the mathematical genius of present times, Albert Einstein, has been quoted saying: "If the facts don't fit the theory, change the facts". The corollary of this maxim is that theory will prevail when facts are inconsistent or difficult to ascertain. Leonardo was thinking along similar lines in the statement: "Those who love factuality (*pratica*) and disregard theory are like the helmsman without a compass and a rudder" [8]. The theory underlying Leonardo's drawing is that the "ideal man" (*homo bene figuratus*) will fit simultaneously in a circle and a square when the arms are stretched out, so that the middle fingers are aligned with the crown of the head, and the legs are opened to an extent to form an equilateral triangle. Therefore, in Leonardo's theory the proportions of the human body are adjustable parameters to satisfy the conditions imposed by the design of the drawing.

3. **The Equilateral Triangle, the Left Foot and Leg Rotation**.

As there are no indications that Vitruvius addresses the relationship between the lowering of the height of man and the angle formed by the opening of the legs in Book III of the Treatise on Architecture and there are no records of related studies in Leonardo's extant notes, the logical deduction is that the idea must have originated specifically as a design feature of the drawing. Although the image in a closed-legs posture would have met the anthropometric requirements of Vitruvius canon, alongside his own data, and would have also satisfied his peculiar condition for the alignment of the finger tips with the head-crown for the arms in the raised position, the design would not have been artistically attractive and would have been physically incoherent without a similar dual position for the legs. The open legs posture, therefore, can be regarded as a component to illustrate the human proportion in a dynamic situation. The introduction of the tangential disposition of the muscles depicted in Figure 3 for the upper delineation of the left leg would allow him to emphasize the rotation of the left foot from the original right-angle position. The addition of a third protrusion, corresponding to the bulge below the waistline on the right side of the image, is to emphasize the convergence of the tangential line at the midpoint of the collarbone, so that in turn it will form a 90^o angle with the upper contour line of the arm. This is not a requirement for the left side where the angle is within the range 82 - 84 ^{o} due to the asymmetry imposed by the formation of the equilateral triangle for the space between the legs, which results in a slight inclination of the base, as shown in Figure 4 (left).

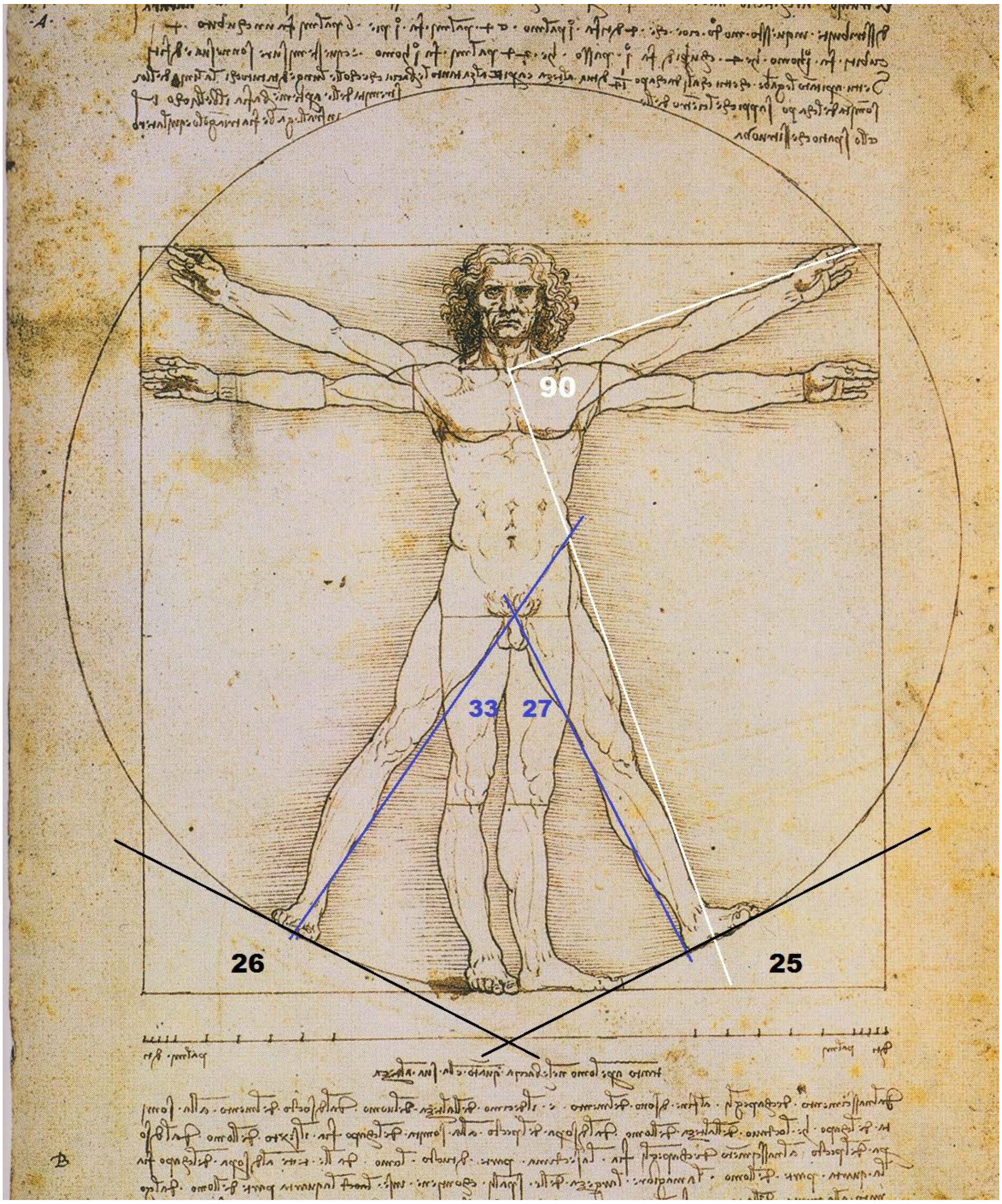

Figure 3. Goniometric data (degrees) and tangential delineations of the contours of Leonardo's drawing.

A scientific interpretation for the right-angle position of the foot in the square of Leonardo's drawing would regard the concurrent angulated posture as a biomechanical requirement for the foot to rotate. The related movements are presented in Figure 4 (left) and represented as a vector diagram in Figure 4 (right), which he may have intuitively used as a concept to account for the discrepancy in the angles swept by the two legs. It can be argued, however, that Leonardo choses the angulated posture of the foot on the right side of the drawing to alleviate the unbalance created by the angular dissymmetry inherent to the opening of the legs, which can be inferred from the observation that the uppermost protrusion of the toes on the two sides have been placed at the same altitude.

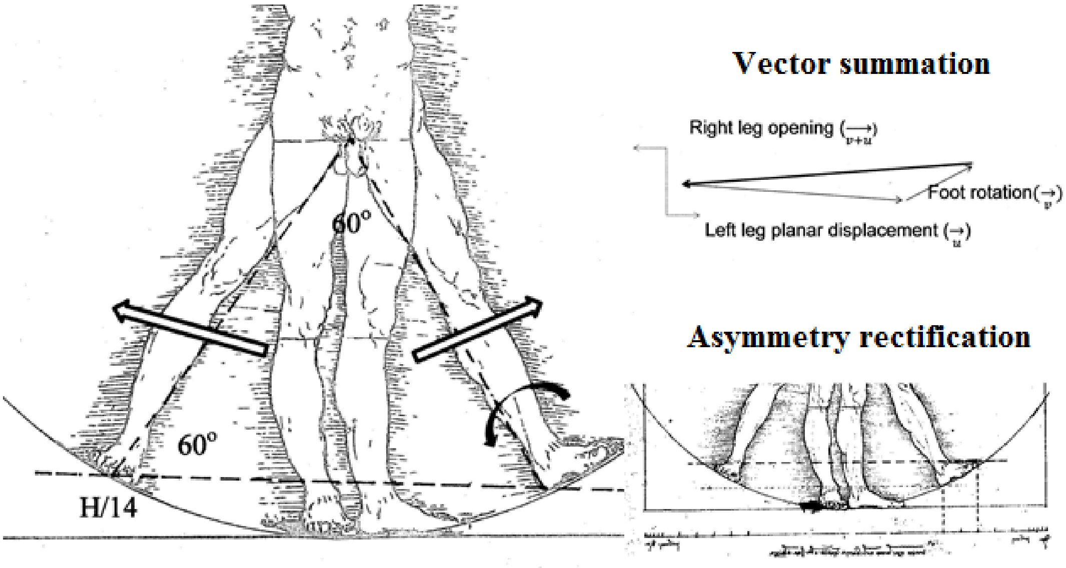

Figure 4. Left: Planar opening of the two legs with angular rotation of the left foot and resulting inclination of the base [see Endnote 2].

Right (top): Vectorial representation of the movements of the legs and retardation angle for the planar opening of the left leg relative to the right leg. Right (bottom): Alignment of upper contours of toes to redress the asymmetry arising from the difference in the amplitude of the displacement of the two legs

It is interesting to note also that the projections of the outer contours of the heel and the toes fall on the 2 palms marks of the measuring scale, as shown in Figure 4 (right bottom), while the tangents to the soles of the feet converge to the centre of the measuring scale (See Figure 3).

……………………………………………………………………………………………..........

<u>Endnote 1</u>: From a modern mathematics perspective Leonardo was seeking intuitively a practical solution to the problem of estimating the position of the navel (radius of circle, R) and the altitude of the collarbone (k) to obtain the interception points of the two circles (see Figure 2 -left). These correspond to the unknowns for the two simultaneous equations.

The general equation of the circle is $(x- h)^2 + (y- k)^2 = r^2$. In this case the co-ordinates of the two parameters are respectively $h = 0$, $r = R$ and $k = R$ for the large circle centred at the navel, and $h = 0$ and $r = H/2$ for the circle that delineates the rotation of the arms. Setting x^2 as the equal term one obtains the equation $2yk – 2yR – k^2 + H^2/4 = 0$. Substituting the two empirically derived parameters, respectively H = 24 *palms* and k = 20 *palms* (i.e. 5H/6), for condition y = H the solution produces an H/R ratio equal to 1.636, which is quite close to the directly measured value, while the small discrepancy can be attributed to the approximation inherent to the H/6 parameter. A much more accurate value would be 9H/48.

Practical measurements on copies of the drawing show that the centre of rotation for the upper delineation of the arms falls just below the mid-collarbone point (D), corresponding to an estimated value of k = 19.5 *palms* (i.e. 24 – 4.5) and goes through the two extreme points of the central horizontal mark (C) at 5H/6 altitude where k = 20 *palms* (i.e. 24 - 4). This is illustrated in Figure 5 where these parameters are confirmed by the incorporation of measuring units in *palms* and *digits* cut out from the measuring scale at the bottom of the drawing.

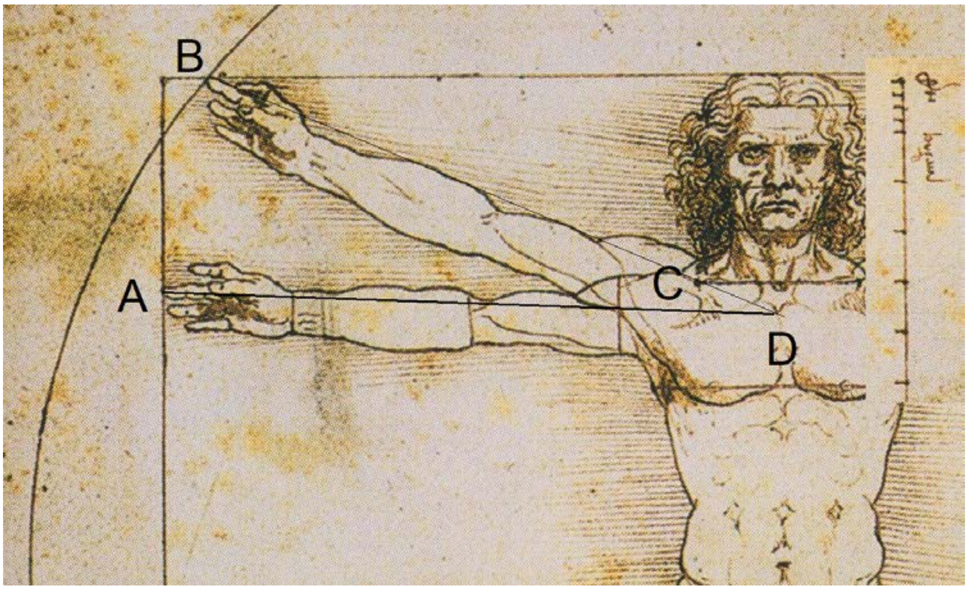

Figure 5. Identification of points, C and D, used as centres for the rotation of the arms to obtain the contact of the middle finger with the square at points A and both the square and the circle at point B.

Using points C and D as the two respective centres of rotation of the arms, the intersections on the circle occur at the same points within the accuracy achievable manually with a compass (see Figure 3, upper left). The measured angles tended by the lines from point D to A and from C to A are respectively $\theta_1 = 20^o$ for the radius AD = BD (R_1) and $\theta_2 = 23^o$ for the radius AC = BC (R_2), while the corresponding lengths are R_1 = H/2 and R_2 = 13H/30 (calculation by Takashi [7]). The equality of cord lengths (AB) is satisfied by the condition $R_1 \cdot \sin \theta_1/2 = R_2 \cdot \sin \theta_2/2$, from which is obtained the solution 0.0868 $\approx$ 0.0863 through substitution of the stated values for the two geometric parameters.

Having identified the contact of the middle finger with the circle from point D, using the procedure described in relation to Figure 2, Leonardo would have introduced this horizontal segment for the distance from the crown of the head as the nearest simple fraction (H/6), which required raising the altitude of the identifying marker. The length of this horizontal segment would have been determined to locate the centre of rotation of the arms, which is more realistic in biomechanics terms, and would provide him with an anthropometric parameter for the distance between the shoulder joints.

……………………………………………………………………………………………………..

Endnote 2: The sagitta (S) of the arc formed by the base of the equilateral triangle with its upper vertex at the pubic bone (centre of the square) is equal to ~ 0.101 R (calculated using the equation S = R (1- cos 26°), where R is the radius of the arc centred at the navel. Since the measured height divided by the radius of the circle is ~1.64 the calculated S/H ratio is approximately equal to ~ 1/16. In Leonardo's drawing the S/H ratio varies respectively from ~1/14 at the lower vertex of the triangle for the contact of right foot to a value varying between 1/16 and 1/18 for the left foot. Note that 26° value is the corresponding angle formed by the tangents of the two feet shown in Figure 3 and half the vertex angle of the isosceles triangle at the navel for a heptagonal inscription of the man in the circle [See Ref 1], i.e. the angle formed by the tangent to the sole of the feet is congruent with angle $\frac{1}{2} \cdot (\frac{2\pi}{7})$.

……………………………………………………………………………………………………..

References

1. L.L. Mascia, A Vitruvius Inspired Criterion for the Construction of Polygons, *Nexus Network Journal* vol 18 no 2 (2016) pp 553-545.

2. H. M. Morgan, *Vitruvius: The Ten Books on Architecture*, Dover Publications, New York, 1960, p 72.

3. M. Fernandez, Da Leonardo a Barbaro: Lettura Grafica dell'Uomo Vitruviano, *Disegnare. Idee immagini*, vol 16, no 2 (2005), pp 10-19.

4. J.P. Richter, Ed. 1970, *The Notebooks of Leonardo da Vinci*, 1883 Reprint, Phaidon, New York. Vol 1, p11.

5. P.A. Torrini, *L'Uomo Vitruviano: dalla Sfortuna alla Fama Odierna* in 'Approfondimenti sull'Uomo Vitruviano di Leonardo da Vinci (Salvi Paola Ed.)', C.B. Edizioni, Milano, 2014, p 79.

6. F. Manenti-Valli, *Per una Rivisitazione Matematica dell'Uomo Vitruviano* in 'Approfondimenti sull'Uomo Vitruviano di Leonardo da Vinci (Salvi Paola Ed.)', C.B. Edizioni, Milano, 2014, pp 88-110.

7. I. Takashi, "Vitruvian Man" by Leonardo da Vinci and the golden ratio. http://www.crl.nitech.ac.jp/~ida/education/VitruvianMan/

8. J.P. Richter, *op. cit.* p18.